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Optimal payout policy under an integrated model of the financial decision calculus of the corporation – Part 1

by

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ABSTRACT

The Dividend Irrelevancy Proposition appears to mean that it is not possible to find an optimal payout ratio. But there is an important exception. If the corporation is growing at a constant rate, and the management forswears recourse to external equity finance, its retention ratio will be defined by its optimal investment strategy. The payout ratio is the complement of the implied optimal retention ratio.

The present paper develops a model that maximises market value added by determining an optimal gearing ratio for the corporation, and its optimal periodic capital expenditure outlays. A steady state growth path is established in accordance with which each of the corporation's accounting and stock market magnitudes grows at a constant rate, and each of its accounting and stock market ratios, including its payout ratio, remains constant over time.

Although the payout ratio does not have the status of a decision variable of the model, the following claims at least may be made for it:

1. it results from an optimisation process
2. it provides a valid prediction of how the well-managed corporation may be expected to behave in relation to its distribution policy: the corporation simply pays out that portion of its earnings it can afford after prioritising its investment spend each period.

The central simulation then shows how the payout ratios of corporations with different growth rates will trace out a U-shaped curve. The slowest and fastest growing companies will have the highest ratios, including 100% in the case of zero growth, of course. In between, the payout ratio dips to about one third.

Generally, the faster the corporation grows, the more it will want to invest, and the greater the proportion of its earnings it will need to retain to finance this process. However, the fastest growing corporations will trade on very high price–earnings (P–E) multiples, and have correspondingly high market capitalisations. This supports a correspondingly high proportion of debt in the financing mix, which reduces the need for internally generated funds, and this in turn enhances the distribution potential. The model then demonstrates how the fastest growing companies might indeed be inclined to pay out their earnings in full.

Each of the corporations in the simulation adopts the same gearing ratio (debt to enterprise value), and is priced by the stock market to generate the same total rate of return: this is the equity cost of capital, which is also an output of the model.

An extended summary of the study is appended to Part II.

INTRODUCTION

'Concentration on this special case appears to be largely responsible for the widely held view that, even under perfect capital markets, there is an optimum dividend policy for the firm that depends on the internal rate of return. Such a conclusion is almost inevitable if one works exclusively with the assumption, explicit or implicit, that funds for investment come *only* from retained earnings. For in that case *dividend policy* is indistinguishable from *investment policy*; and there *is* an optimal investment policy which does in general depend on the rate of return.'

Apart from the definition in the quotation itself, the special case under discussion here is that of the corporation that progresses along a steady state growth path – that is, all its accounting and stock market magnitudes are growing at a uniform rate – and the discussants are of course Merton Miller and Franco Modigliani (1961, their italics). It might not be too fanciful to suggest that they are not entirely pleased to have to make this concession in relation to their Dividend Irrelevancy Proposition, and to drive the point home, their Figure 1 shows the corporation that relies exclusively on retained earnings for investment funds as one of two extreme cases. At the other extreme is the corporation that pays out its entire earnings as dividends: it is then forced to fund the same 'optimal investment policy' by means of a series of equity financings. Stockholders in the two corporations earn the same total rate of return: the higher dividend yield on the corporation that operates the 100% payout ratio is exactly offset by the slower rate of dividend growth caused by the dilution effect. Of course, any intermediate combination of payout ratio, yield and growth is equally possible, provided the latter two figures sum to this same rate of return – which is, in fact, the equity cost of capital.

Miller and Modigliani vehemently dispute the relevance of the constant growth, self-financing paradigm, but nearly 50 years after their article was written most standard texts on financial management place it in the ascendancy among the tools available to investment analysts and other practitioners concerned with valuing companies. In many cases, analysts are able to make explicit forecasts of earnings and dividends for the upcoming few years, and these forecasts may indeed manifest different rates of advance from year to year. But corporations have infinite lives – at least as far forward as the mind can envisage – and from some point in the future the only way to handle the valuation challenge is to compute a residual value using the dividend discount model, or some derivative thereof, assuming a constant rate of growth through to infinity. Consistent with the constant growth model is the fact that the corporation's retention ratio – the complement of its payout ratio (together the pair must add to 100%) – will be equal to the ratio of its growth rate to its net (of corporation tax) return on equity, as noted by Modigliani and Miller in the quotation.

This relationship fails to provide a valid decision rule for the payout ratio, however. The corporation's return on equity cannot be calculated without first determining its optimal gearing ratio – and indeed its optimal periodic capital expenditure outlays – as will become evident in due course. It is, in fact, these magnitudes that represent the decision variables of the model. But here again the story rubs up against the Modigliani–Miller canon. In the latter of two articles on financial policy (1958, 1959), they argue that, under a corporation tax system that allows the interest on the corporation's debt as an expense, the value of the corporation will be optimised by gearing up to the level where, if possible, the whole of the capital expenditure plan is financed each year by incremental borrowing. One fortunate consequence of this result, of course, is that the corporation can operate a 100% payout ratio without antagonising its stockholders with a never ending succession of equity offerings. Both the UK and the US have such corporation tax systems.

An alternative theory of the optimal gearing ratio is put forward by Brealey, Myers and Allen in their text, *Corporate Finance* (2006). They questioned whether the Modigliani–Miller theorem gives sufficient weight to the possibility of financial distress, and formulated their *Trade-Off Theory of Capital Structure* accordingly. This shows the value of the corporation at first rising as a function of the gearing ratio as the benefit of the tax shield dominates: but as the discounted future costs of financial distress mount up in response to the increasing probability of default, the curve rolls over, and the value of the corporation declines. The present study develops an integrated model in which the corporation's optimal gearing ratio is determined according to the Brealey, Myers and Allen theory. Simultaneously, the model solves for the corporation's optimal periodic capital expenditure outlays. The gearing ratio is constant through time: the amount to be invested year by year increases at the corporation's underlying growth rate. Its growth rate is determined by the rate at which its marginal efficiency of capital expenditure curves shift to the right over time.

Although the payout ratio is not a decision variable of the model, it is optimal in the sense noted by Modigliani and Miller – as the direct consequence of implementing the corporation's optimal periodic capital expenditure plans, and relying only on internally generated equity funds. The solution does imply that the corporation will raise new debt finance each year. The relationship of interest is then to demonstrate how the payout ratio varies between companies with different rates of growth. A mathematical formula is derived which, firstly, indicates a payout ratio of 100% for the zero growth case. Thereafter, as the rates of growth of the corporations increase, the payout curve falls away to a minimum. Finally, at the faster rates of growth, the formula turns and begins to rise, eventually registering 100% again.

The formula has important consequences for the way chief financial officers (CFOs) approach the problem of making recommendations on year-to-year dividend payments to their boardroom colleagues. The model also generates the trade-off between growth and the price–earnings ratio, which will expand the resources available to investment analysts in their attempts to put a value on the corporation's shares. A further interesting by-product of the analysis is the behaviour of the effective tax rate, which is seen to be a declining function of the growth rate. The tax rate suffers a chronic collapse in the case of the fastest growing corporation in the simulation, thus completing a virtuous circle in the valuation process.

Plan of the Study

The remainder of this introduction provides further guidance on the intended progress of the discourse. The study is published into two parts: Part I falls into the seven sections outlined below.

I. **Building Blocks.** The initial section develops the accounts for a zero growth corporation. This permits the concepts required for the analysis of growing corporations to be introduced in the context of a simpler framework than would otherwise be the case. Figures are described that focus on the weighted average cost of capital (WACC) for the corporation, and the marginal efficiency of capital expenditure (MEC). The basic valuation model is the dividend discount model: this reduces to the formula for a perpetuity in the zero growth case. In addition, in preparation for the analysis of growth, a figure demonstrating the dynamic of the MEC curves facing the corporation over time is presented.

II. **Maximising Market Value Added.** There follows a section detailing the construction of a neo-classical micro-economic optimisation model. An equation is developed for the market value added (MVA) of the corporation as it depends on two decision variables, the debt ratio and the capital expenditure outlay at the end of accounting Period One. The techniques of the differential calculus are engaged to establish formulae which may then be solved for the optimising values of the decision variables.

III. **The Optimal Gearing Ratio.** Section III returns to the issue of the corporation's WACC, as the previous analysis now permits a more detailed explanation of the earlier figure. The marginal conditions for the minimisation of the corporation's cost of capital are established, which in turn has implications for the slopes of the equity cost of capital (ECC), and for the interest rate on the corporation's debt, as these relationships depend on the ratio of debt to enterprise value.

IV. **The Optimal Level of Period One Capital Expenditure.** Similarly, it is necessary to demonstrate the meaning of the decision rule for the corporation's investment spend for the period: this is that it should invest up to that amount where the MEC curve falls to equality with the WACC, which, by this stage in the analysis, may be treated as a constant – that is, invariant with time or the rate of growth. The corporation's capital expenditure amounts then grow steadily at a rate determined by the period-to-period shifts in its MECs.

V. **Central Simulation.** This section goes on to detail the computations necessary to derive a full set of accounts for the corporation growing at 6% per annum. Comparable calculations apply to other corporations growing at other rates, and this permits the completion of a table (Table I-1 below) detailing the balance sheets and profit and loss accounts of corporations with growth rates between 0% and 9% per annum.

VI. **A Picture of the Payout Ratio.** The main interest of the table is in the behaviour of a number of accounting items and accounting ratios and stock market ratios as they depend on the rate of growth. Each of the corporations is priced in the stock market to generate the same total rate of return – as it works out, 10.25% – and of particular interest then is the extent to which growth justifies paying a higher multiple of earnings, and how the optimal payout ratio falls, and subsequently rises, as a function of the growth rate.

VII. **The Story So Far.** The study assumes throughout that the corporation makes its distributions by repurchasing its own stock in the market: this is a matter of efficient tax planning in relation to the interests of those stock holders who are liable to the higher rate of income tax. This policy has the advantage that it clearly exposes the inherent circularity of the Dividend Irrelevancy Proposition. Once the contra-entries have been set aside, it immediately becomes apparent that there is a meaningful relationship between the rate at which the corporation grows and the optimal payout ratio.

Part II of the study is published as a companion piece to the present paper. The main outstanding task will be to reconstruct the diagram used by Brealey, Myers and Allen (2006) to illustrate their Trade-Off Theory of Capital Structure by examining the financial characteristics of corporations that fail to adopt their optimal gearing ratios. It will also expand on the behaviour of a number of magnitudes and ratios as they depend on the rate of growth of the corporation, most importantly the market value added ratio, as it is the MVA which serves as the optimand in the model developed in Part I. References will be listed at the end of Part II.

I. BUILDING BLOCKS

Methodology

The subsequent section of this paper sets out a mathematical model of the corporation. An equation is developed for the corporation's market value added, as it depends on its decision variables – its gearing ratio and its periodic capital expenditure outlays. By differentiating this equation with respect to these variables, conditions for the maximisation of the market value added are established. The model is then applied to corporations growing at rates between 1% and 9% per annum. The results are shown in Table I-1, together with comparative data for the case of zero growth. The simulated corporations in Table I-1 illustrate the relationships between a number of financial magnitudes – price–earnings (P–E) ratio and payout ratio among them – as they depend on the rate of growth. These relationships are presented in a set of graphs in later sections of the study.

Before developing the main model, it is instructive to deal with the case of zero growth. This permits a preliminary exploration of the basic building blocks of the model: the determination of the corporation's weighted average cost of capital (WACC), its marginal efficiency of capital expenditure (MEC), the role of the dividend discount model, and the forces driving its growth.

Market Value Added (MVA)

Market Value Added is defined as the difference between the enterprise value of the corporation and the book cost of its assets: enterprise value is the sum of its equity market capitalisation and its outstanding debt. In the situation where its debt trades at face value, the MVA will equal equity market capitalisation less the balance sheet value of the equity: the latter, in turn, is the accumulation of past retained earnings. Market Value Added is the net present value (NPV) of the future excess profits to be earned by the corporation, where such excess measures the surplus over and above the profits required to generate the market rate of return on the corporation's sources of finance, both equity and debt. The concept of MVA is examined by G. Bennett Stewart III in *The Quest for Value* (1990), and he details the many accounting adjustments, both to profits and to assets, which need to be made to the data appearing in conventional accounts before it is possible accurately to compute return on capital employed. Year by year, the excess profit is given by the product of capital employed and the difference between the return on capital employed and the WACC as defined below.

Market Value Added serves as the optimand in the present study, and in due course it will be possible to demonstrate why the maximisation of enterprise value, or the corporation's price–earnings ratio, can lead to the impairment of shareholder value. Table II-2 in Part II of the study simulates the accounts of a corporation that over-expands in such a way as to boost its enterprise value, but that, in so doing, suffers a diminution in its MVA.

Weighted Average Cost of Capital (WACC) – Building Block 1

Brealey, Myers and Allen, in their *Corporate Finance* (2006), develop their Trade-off Theory of Capital Structure. They postulate that the beneficial effects of the tax shield on the interest on debt will be balanced and ultimately overwhelmed by the present value of the costs of financial distress. Their theory is illustrated in a graph showing the market value of the equity and debt together against the ratio of debt to enterprise value. Market value initially increases with the debt ratio, but subsides again when the probability of financial failure becomes too burdensome. They argue that the rate of interest faced by the corporation will increase with its rising probability of default. This effect is shown in Figure I-1, in which the relationships are drawn up on a post corporation tax basis.

The graph shows (the thicker lines) the rate of interest (this and similar variables are expressed in decimal fractions on the vertical axis) paid by the corporation on its debt (INTRTE – the squares), and the equity cost of capital (ECC – the circles), as they depend on the gearing ratio (d) on the horizontal axis. Equity investors too are risk averse, and bid the stock onto a higher total rate of return as earnings become more volatile with increasing debt. The curve labelled WACC (the diamonds) is then the market value weighted average of the INTRTE and ECC relationships:

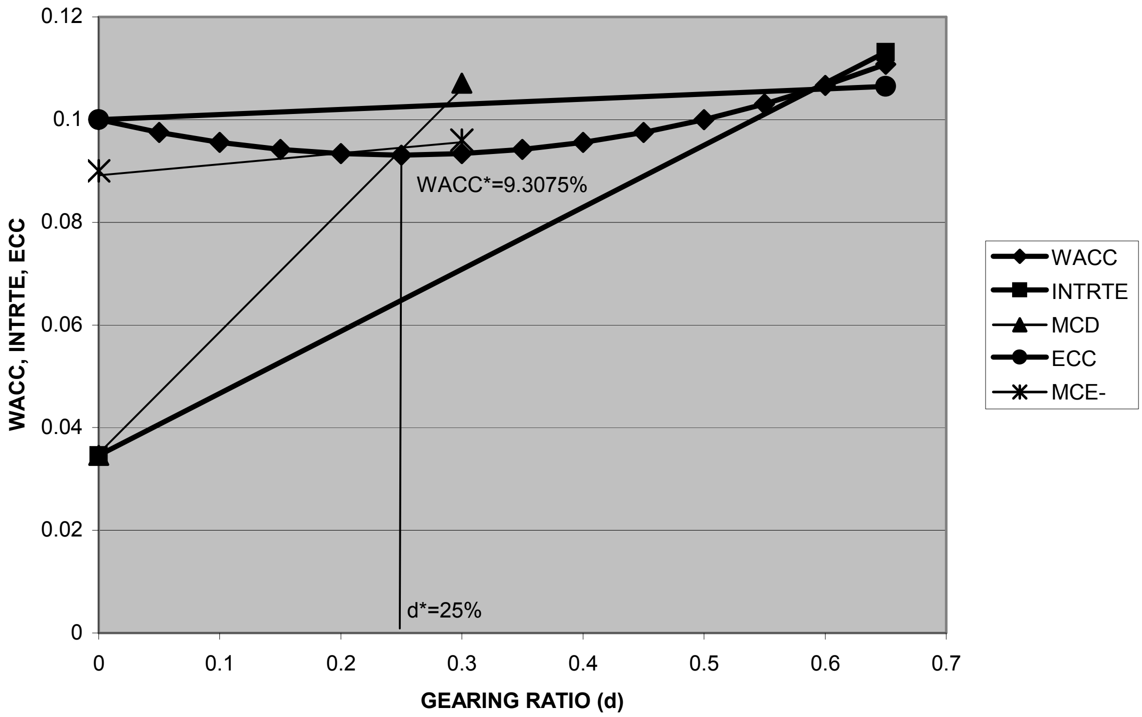
$$\mathrm{WACC} = \mathrm{dn(d)} + (1 - \mathrm{d})\mathrm{r(d)}$$

where the weights of the debt and the equity are, respectively, d and (1 – d), n(d) is used for the mathematical representation of the interest rate function, and r(d), that of the ECC. Detailed mathematical formulae will be introduced in later sections of the paper – in fact, with the purpose of keeping the analysis as simple as possible, the INTRTE and ECC functions are assumed to be straight lines – but, in the meantime, the graph is drawn so that the WACC reaches its minimum where the debt ratio takes the value 0.25 (25%): an asterisk is used to denote the fact that this is the optimal gearing ratio. At this level of the debt ratio the interest rate works out at 6.48% (this is the rate after corporation tax), and the ECC, at 10.25%. The minimal value of the WACC is then 9.3075% – that is:

$$\mathrm{WACC} = 0.25 * 0.0648 + 0.75 * 0.1025 = 0.093075$$

as indicated in Figure I-1.

FIGURE I-1



This value for the WACC will be used in a simulation demonstrating the financial characteristics of a static corporation: each of its accounting and stock market magnitudes remains constant through time. It will invest only to maintain the productive capacity of its existing assets, and such investment will be financed by its annual depreciation provision. It will thus adopt a 100% payout ratio.

The thinner lines in Figure I-1, including the marginal cost of debt (MCD), will enter the discussion in Section III below.

Corporation Tax and Personal Taxes

It is difficult to proceed very far in finance without encountering the problems thrown up by taxes of one sort or another. In the case of interest, the problems are mitigated by the fact that interest is generally treated as an allowable expense when calculating the amount of profit subject to corporation tax: this gives rise to the tax shield effect. In particular, this arrangement means that it is sensible to compute project rates of return at the net of corporation tax level: the gearing ratio and the associated interest charge can then be accounted for as a separate exercise. The UK corporation tax rate may now be taken as 28% (the rate was reduced from 30% in the 2007 Finance Act), which means that the net interest rate of 6.48% quoted above translates to a gross 9% (= 6.48/(1.00 – 0.28)) paid to the bondholders. Of course, some bondholders may be subject to income tax, but generally bonds are more attractive to gross funds – such as the pension funds – which in the UK are subject neither to income tax nor to capital gains tax. If there is this type of clientele effect, it is not something the corporation's managers need to be too concerned about in their decision taking. The tax shield effect will be illustrated in due course.

The main tax contrast is between the gross funds, which pay no tax as noted, and wealthy private individuals. Private individuals investing in equities are faced with paying both income tax and capital gains tax, and this difference in treatment does potentially pose a significant problem in relation to developing an optimisation model for the corporation. However, in recent years, both in the UK and the US, corporations have been free to distribute their surpluses by repurchasing their own shares in the marketplace, rather than by declaring cash dividends. When they do so, the liability to income tax is avoided by their private stockholders. Other things being equal, the ongoing stockholder will enjoy a capital gain equivalent to her share of the buy-back: effectively, her stock will become 'full of dividend', but instead of going ex such dividend, it will sustain the higher value to reflect the fact that she will own a greater proportion of the corporation after other stockholders have sold out. Of course, this additional gain on her investment in the corporation will imply a greater capital gains tax liability, but this does not involve any actual payment until she decides to dispose of her stock. Even then, in the UK, individual annual and other allowances may be employed to mitigate the incidence of the tax on gains.

For those investors paying higher rate income tax, the rate on cash dividends works out at 25% (this applies to the net dividend received): at the margin, gains will in future (from 2008/9 onwards) suffer tax at 18%. There is then a contrast with the circumstances of those investors paying the standard rate of income tax at the margin: such investors will be subject to no further liability in relation to cash dividends received, but capital appreciation may in their hands suffer capital gains tax at the proposed rate of 18% also. At present dividend tax and gains tax are both assessed at the rate of 15% in the US, but this lower rate of dividend tax (as compared with the rate in the time of the previous administration) remains to be confirmed by the Congress in 2010. Maximising MVA then entails assuming that the corporation makes its distributions in the form of stock a sequence of repurchase programmes, rather than by paying cash dividends. Making a distribution via a cash dividend will tend to depress the value of the corporation to reflect the incidence of income tax: the income tax can then only be avoided by selling the stock to invest in a company that adopts the repurchase route.

Efficient tax planning thus entails the combination of the corporation distributing its dividend pot via a sequence of repurchases, and the stockholder sitting on her accumulating capital gains. This means that personal taxes may be ignored for analytical purposes. The Modigliani–Miller assumption of investor rationality proves supportive here: the gross funds are not disadvantaged by a procedure which benefits wealthy individual private stockholders. Thus, managers can ignore personal taxes in their decision taking, even though some stockholders may from time to time find it beneficial in relation to their individual interests to realise some of their holdings, and so possibly to pay away some of their accumulating appreciation in capital gains taxes.

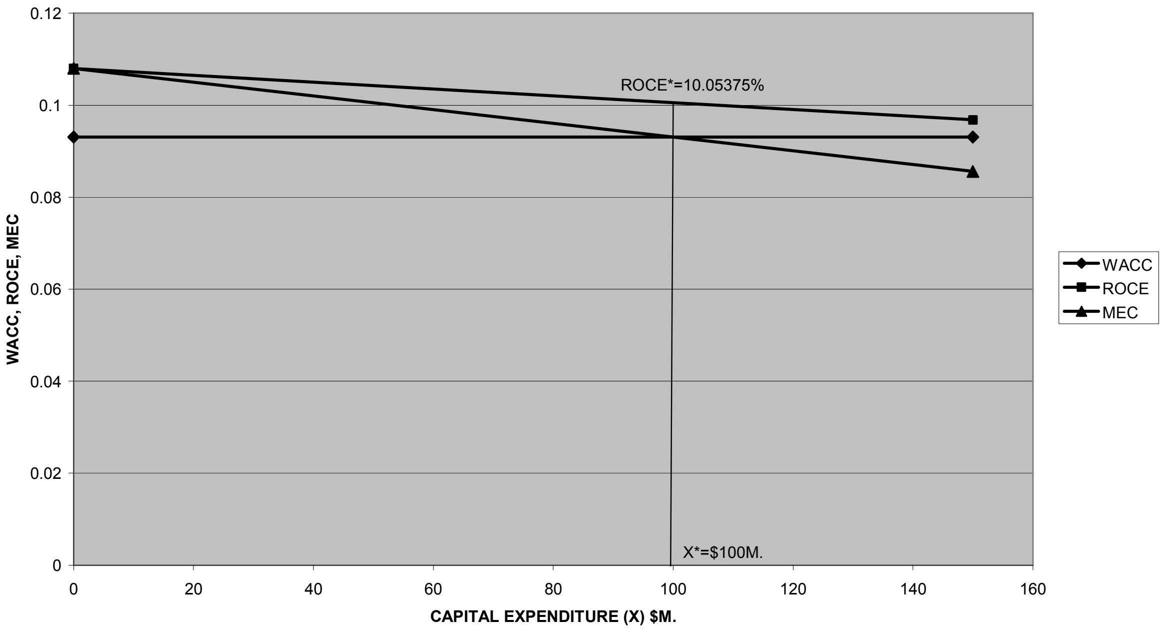
The Marginal Efficiency of Capital Expenditure (MEC) – Building Block 2

The objective in Section I is to illustrate the calculation of the MVA of a corporation manifesting no growth at all: to some extent, this is the base case in the simulations detailed in Table I-1. For the purposes of the simulation, it may be assumed that a new corporation is being formed, and it has to decide how much capital to raise, debt and equity (now, in the light of the analysis of Figure I-1, in the ratio of one to three), to lay out on its (once for all) investment spend. The corporation is assumed to have identified a number of possible projects, each with the profile of a single cash outlay, followed by a steady annual flow of post corporation tax profits commencing one year later. Profit is assumed to be struck after the corporation has provided for depreciation, and it is further assumed that it will expend any such amounts to maintain the productive capacity of its assets over an infinite life (a 10% depreciation rate is introduced explicitly in a Table I-2 later). It is further assumed that there are no cash flow effects resulting from timing differences between accounting depreciation and the depreciation charged for tax purposes, and that the cash flows on the various potential projects are independent of one another.

The prospective internal rate of return on each project is thus the ratio of its forecast net profit to its associated capital expenditure outlay, and the analysis begins by ranking the projects by these rates of return. It is then possible, starting with the most profitable project, to cumulate both profits and the amounts invested, and to take the ratio of the one to the other. This generates a relationship for the average rate of return on the first n (say) most profitable projects, and it is this relationship that provides the framework for the function labelled ROCE (return on capital employed – the squares) in Figure I-2 overleaf. In practice, projects tend to vary in size, but it is convenient for the purposes of the analysis to iron out any such lumpiness, and to represent the relationship as a continuous straight line. On the horizontal axis, X represents the possible amounts the corporation could raise and spend on its projects: on the vertical axis, the relationship shows the corresponding average rate of return the corporation will earn on its investments. These returns are expressed in terms of decimal fractions, and, as drawn, the line indicates that the highest earning project has an internal rate of return 10.8% (0.108): this equates to 15% (= 10.80/(1.00 – 0.28)) before corporation tax.

If, as proposed, the ROCE is represented as a straight line, it will be the case that the relationship for the total (cumulative) profit function (as discussed earlier), will follow the curve of a quadratic equation, rising up from zero where capital expenditure, X, equals zero, reaching a maximum, and then falling back to zero again where the ROCE crosses the horizontal axis (the latter point is not shown in the figure). The total profit curve may be written mathematically as Xf(X), the product of the initial capital expenditure, X, and the ROCE, which will be denoted mathematically by f(X) in the following section of the paper.

FIGURE I-2



The derivative – that is, the gradient – of the curve for total net profit is shown as the relationship denoted by MEC (the triangles) in Figure I-2: it is the marginal efficiency of capital expenditure, and shows, as a function of the amount invested, the increase in its total profit – this will be measured in cents per annum – the corporation will generate for a $1 addition to its capital expenditure. Copeland and Weston (1992) refer to this curve as the 'marginal efficiency of investment': Keynes (1936), working at the macro level of course, made use of the concept of the marginal efficiency of capital in the General Theory. The total profit function reaches its maximum for that value of X where the MEC crosses the horizontal axis: at this point, the slope will be zero – the extra dollar invested will add nothing to the annual profit count. If, as assumed, the ROCE is a falling straight line, the MEC will also be a falling straight line: it will take the same value – 0.108 – on the vertical axis (where X = 0), but will slope downwards at twice the gradient. As noted, the straight line assumption makes the mathematics simpler, but more complicated relationships could equally be assumed. As drawn, the ROCE has a (negative) gradient of 0.000074625: the MEC thus has a (negative) gradient of 0.00014925.

The WACC relating to the optimal gearing ratio was calculated in the earlier sub-section. The WACC is shown as the horizontal line at the value of 0.093075: it depends on the relationships in Figure I-1, but is assumed to be independent of time and of the corporation's scale of operation, X. Here the decision rule is that the corporation should set its level of capital expenditure, X, at that amount such that the MEC falls to the level of the WACC. The corporation should only invest in projects with a positive net present value: if a project fails to cover the WACC, it should not be undertaken.

As drawn in Figure I-2, the MEC cuts the WACC above the point on the horizontal axis where capital expenditure takes the value $100.0m: again an asterisk is used to denote the optimal level of X. The average rate of return on investment, f(X), is then 10.05375% (= 100*(0.108 – 100*0.000074625)): this is labelled ROCE*, and it is, as a matter of geometry, the simple average of the intercept, 0.108, and the WACC, 0.093075. Investing $100.0m generates an annual profit, after corporation tax but before (net) interest, of $10.05375m in perpetuity: Bennett Stewart (1990) refers to this amount as the NOPAT – net operating profit after tax. Grossing up at 28% then indicates a profit before interest and tax of $13.964m. This is the figure at the head of the Profit and Loss Account for the zero growth corporation in the leading column of Table I-1 overleaf, and it is referred to as 'operating profit' therein.

Discounted Cash Flow Model – Building Block 3

The NOPAT does not appear in the P&L account proper (in the table, the rate is present as a memorandum item under 'Accounting ratios'), but it represents the net income generated annually by the corporation on behalf of both its bondholders and its stockholders. In the case of zero growth, it will be the product of capital employed and ROCE* throughout infinity, and may be discounted at the WACC to give the enterprise value (V) of the corporation:

Table I-1 P&L Accounts for Period One (Debt/Enterprise Value = 25%)

	Growth – per cent per annum									
	0	1	2	3	4	5	**6**	7	8	9
Balance sheet (beginning period) ($m)										
Capital employed	100.000	100.000	100.000	100.000	100.000	100.000	**100.000**	100.000	100.000	100.000
Financing										
Equity	72.996	72.754	72.447	72.042	71.485	70.669	**69.359**	66.915	60.731	14.329
Debt	27.004	27.246	27.553	27.958	28.515	29.331	**30.641**	33.085	39.269	85.671
Profit and loss account ($m)										
Operating profit	13.964	13.964	13.964	13.964	13.964	13.964	**13.964**	13.964	13.964	13.964
Interest	2.430	2.452	2.480	2.516	2.566	2.640	**2.758**	2.978	3.534	7.710
Profit before tax	11.533	11.511	11.484	11.447	11.397	11.324	**11.206**	10.986	10.429	6.253
Corporation tax	3.229	3.223	3.215	3.205	3.191	3.171	**3.138**	3.076	2.920	1.751
Net profit	8.304	8.288	8.268	8.242	8.206	8.153	**8.068**	7.910	7.509	4.502
Retained profit	0.000	0.728	1.449	2.161	2.859	3.533	**4.162**	4.684	4.859	1.290
Payout	8.304	7.561	6.819	6.081	5.347	4.620	**3.907**	3.226	2.651	3.213
Accounting ratios (%)										
ROCE	10.054	10.054	10.054	10.054	10.054	10.054	**10.054**	10.054	10.054	10.054
Return on equity (net)	11.376	11.392	11.413	11.441	11.479	11.537	**11.633**	11.821	12.365	31.420
Payout	100.000	91.222	82.476	73.778	65.155	56.661	**48.420**	40.782	35.299	71.356
Interest cover	574.537	569.449	563.098	554.945	544.100	528.962	**506.356**	468.944	395.100	181.101
Stock market statistics										
Equity capitalisation ($m)	81.013	81.737	82.659	83.873	85.545	87.993	**91.922**	99.255	117.806	257.012
Enterprise value ($m)	108.018	108.983	110.212	111.831	114.060	117.324	**122.562**	132.340	157.075	342.683
MVA ($m)	8.018	8.983	10.212	11.831	14.060	17.324	**22.562**	32.340	57.075	242.683
P–E ratio	9.756	9.862	9.997	10.176	10.425	10.793	**11.393**	12.548	15.688	57.085
Disbursement yield (%)	10.250	9.250	8.250	7.250	6.250	5.250	**4.250**	3.250	2.250	1.250
Overall tax rate (%)	23.127	22.935	22.696	22.385	21.969	21.380	**20.484**	18.956	15.760	4.890
PEG	Infinite	9.862	4.999	3.392	2.606	2.159	**1.899**	1.793	1.961	6.343
MEC function										
Intercept (*100)	NA	10.800	10.800	10.800	10.800	10.800	**10.800**	10.800	10.800	10.800
Gradient (*100)	NA	1.493	0.746	0.498	0.373	0.299	**0.249**	0.213	0.187	0.166

$$V = 100.0 * 0.1005375 \sum 1/(1 + WACC)^t = 10.05375/WACC$$

$$= 10.05375/0.093075 = 108.018$$

where the summation runs from t = 1 through infinity, which then results in the formula for a perpetuity. The enterprise value of $108.018m represents the sum of the equity market capitalisation (Q) and the outstanding debt (B):

$$Q = 0.75V = \$81.013m.$$

and:

$$B = 0.25V = \$27.004m.$$

where the gearing ratio is taken as 25% as deduced from Figure I-1. The formula for V will be justified in Section II. Given an initial investment of $100.0m in capital employed – financed in part by equity of $72.996m – it is immediately apparent that the MVA amounts to $8.018m. As noted, this is the present value of the excess profits the corporation will generate over an infinite lifetime, and its modest dimension in this case reflects the rather limited spread between the ROCE and the WACC, only about three-quarters of 1% (= 10.05375 – 9.3075).

Progressing through the annual P&L account, interest at 9% gross on debt of $27.004m amounts to $2.430m, leaving profit before tax of $11.533m. Corporation tax at 28% then claims $3.229m, leaving earnings at $8.304m, all of which is available for paying the dividend, as the static corporation will make no further allocations to capital expenditure. However, in order not to involve those of its stockholders subject to personal taxes in paying income tax – the UK higher rate would be effectively 25% on the cash amount, as noted – this residual net free cash flow should be used for a stock repurchase plan, and is then labelled 'payout' in the table. This represents 10.25% – the same as the ECC calculated above – of the equity market value of the corporation, $81.013m. This is shown as the 'Disbursement yield' under 'Stock market statistics' in the Table I-1.

The Tax Shield Effect

Corporation tax at the rate of 28% on operating profit of $13.964m would amount to $3.910m, if the corporation were financed purely from equity: this would leave a net profit of $10.054m. By borrowing $27.004m at 9%, the corporation incurs an annual interest charge of $2.430m as shown in Table I-1. This amount may be conceived as being drawn from two sources, as compared with the equity financed corporation. Firstly, there is a reduction in the net profit attributable to the stockholders of $1.750m (= 10.054 – 8.304); secondly, there is a reduction in the corporation tax payment of $0.681m (= 3.910 – 3.229). Thus, the tax shield boosts the share of the stockholders and bondholders together in the operating profit to about 76.9% of the operating profit: correspondingly, compared with the 28% nominal rate of corporation tax noted above, the proportion due to the tax authorities is in fact reduced to 23.127%.

This last figure is the overall tax rate shown in Table I-1 for the zero growth case: it is the ratio of the corporation tax paid to the sum of the corporation tax paid (again), the gross interest and the payout. In other cases in the table, this variable will also be affected by the capital appreciation on the corporation's stock. If, as in the case now under discussion, the corporation is not growing at all, there will be no change in its equity market value from year to year.

The Stock Repurchase Programme

However, if it buys in its own shares, its stock price should manifest a steady increase over time: this applies to its reported earnings per share also.

It would hardly make sense to seek the corporation's optimal payout ratio without assuming that the management and stockholders take advantage of the opportunity to avoid the payment of income tax on dividends and the payment of gains tax on capital appreciation. But the repurchase process does have the unfortunate side effect of distorting the corporation's rate of earnings growth, and it is therefore appropriate to examine the arithmetic of the case of zero growth to begin to understand how to deal with this problem.

If, at the end of its fundraising, the corporation has 100m shares in issue, each will stand initially at 81.013c in the marketplace, but by the end of one year, they will have risen by 8.304c ((cash) earnings per share) to 89.317c: this is a rise of 10.25%. 'Cum dividend' equity market capitalisation will stand at $89.317m. The dividend pot will now buy in 9.297m (= 100*8.304/89.317) shares, leaving 90.703m in issue. The corporation's total earnings in the following period (Period Two) will again amount to $8.304m, which now equates prospectively to 9.155c (= 8.304/90.703) per share: this represents a 10.25% (the ECC again) increase over the earnings per share for Period One. Of course, the prospective price–earnings ratio (P–E) remains unchanged, as the stock price never manifests the fall that would otherwise have occurred when the stock went ex dividend: that is, the prospective P–E after the repurchase transaction will again be 9.756 (= 89.317/9.155). The effect of the annual repurchase programme is to exchange a 10.25% dividend yield for a 10.25% increase in the stock price. The figure for earnings per share appears to rise by 10.25% per annum, but this represents the sacrifice of the cash dividend, rather than any growth in the corporation's earnings, which in aggregate remain at $8.304m in perpetuity. To make them comparable, the previous year's earnings (referenced Period One in Table I-1) should be scaled up by a factor equal to the old number of shares in issue to the new number – obviously 1.1025 (= 1.0 + ECC) in this case. This adjustment has the effect of reducing the growth in earnings per share to zero, the same as the net profit.

Over time, a non-participating stockholder will acquire an increasing percentage holding in the corporation, but this is no different from receiving a cash dividend each period, and using it to reinvest in the company – except, of course, in those cases where there would have been leakage due to an income tax liability. The economic value of the holding would be the same: it is just the number of shares that will be different. It is obviously beyond the remit of the management of the corporation to take into account the fact that some taxable stockholders may from time to time have their own reasons for realising some of their stock, and for then suffering any associated capital gains tax liability.

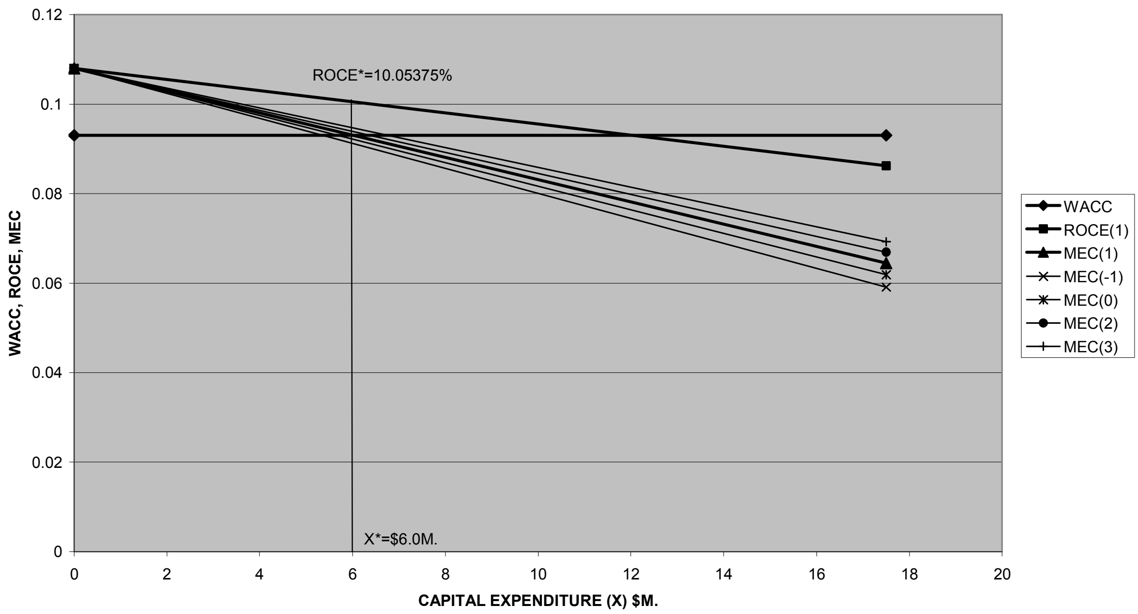
Other statistics for the zero growth case will be examined in the later comparisons, but in the meantime it is worth remarking that the payout ratio shown in Table I-1 – the figure is 100% – now refers to the repurchase programme, which is fully disclosed in the P&L account. Thus it has been necessary to rename the 'yield' on the stock as the 'disbursement yield'. By convention, a corporation would only provide for a cash dividend in this manner: buy-backs would be financed by drawing on the corporation's accumulated revenue reserves, which will have passed from the P&L account as retentions, possibly over a number of years. This treatment often makes it difficult to compute an accurate payout ratio for any given year, especially when the corporation may in practice be making repurchases financed from other sources: by asset disposals, or by issuing additional debt to recalibrate its balance sheet.

Details of the corporation's repurchase activity may not be made fully explicit until the publication of the following year's accounts. The need for some tightening up in the procedures for accounting for stock repurchase programmes will be taken up in the conclusion in Part II of the study.

Dynamic of the MEC – Building Block 4

The other cases in Table I-1 show comparable accounts and other financial data for corporations growing at rates varying between 1% and 9%. Figure I-3 builds on the Figure I-2 construction to depict the process that is driving this growth.

FIGURE I-3



The corporation's growth is determined by an ever-expanding investment opportunity, presumably reflecting a response to the expansion of the economic environment in which it operates. This is modelled as a steady rightward shift in the MEC curves it draws up year by year. The MECs – the sample in the figure refer to the case of growth at 6% – swing to the right like a pendulum suspended at a value of 0.108 (10.8%) on the vertical axis: this means that, each year, the highest possible rate of return (net) the corporation can invest at is 10.8%. But the pendulum swings in the same direction for ever, cutting the WACC – which has been drawn in at a value of 9.3075% as before – 6% further to the right each period, indicating 6% growth in X, the amount to be invested. Looking backwards in time, the MECs become progressively steeper, and thus closer to the vertical axis: but they never become exactly vertical, as the process is assumed to be infinite. As a working assumption, the corporation has an infinite history: obviously, this cannot be the case, but mathematically this makes it possible to establish the corporation's asset base.

MEC(1) is drawn with a (negative) gradient of 0.0024875, so that:

$$\mathrm{MEC}(1) = 0.108 - 0.0024875\mathrm{X}_1$$

This means that, at the end of Period One, MEC(1) cuts the WACC – 9.3075% as calculated earlier – above a value of 6.0 ($6.0m, say) on the horizontal axis, indicating the desired level of capital expenditure for the period. The periodic shift is then such that MEC(0) cuts the WACC at a capital expenditure value of 6/(1.06), MEC(-1) at capital expenditure of

$6/(1.06)^2$, and so on through an infinite regression. Again, the perpetuity formula applies, so that:

$$A_0 = \sum 6.0/(1.06)^t = 6.0/0.06 = 100.0 = X_1/g$$

where the summation extends over the period t = 1 through infinity, and A_0 is now the book value of the corporation's assets immediately after an investment of 6/(1.06) at the end of Period Zero. MEC(2) and MEC(3) result in capital expenditure figures of 6.0*(1.06) and $6.0*(1.06)^2$ respectively, and so on. The first of these results is obtained by solving:

$$\text{MEC}(2) = 0.108 - 0.0024875X_2/(1+g) = 0.093075$$

at the second equality. Dividing through the second term in the MEC by (1 + g) reduces the gradient so that the solution gives a 6% greater investment in Period Two for g = 0.06.

As a further simplification, it will later be assumed that net working capital is zero, so that A_0 may be interpreted as fixed assets. It is assumed that the corporation depreciates its assets (a figure of 10% is used later in Table I-2) and spends the equivalent amount each year on maintaining their productive capacity: it is further assumed that there are no complications arising due to different depreciation allowances being used for computing the tax liability for each period.

Corresponding to MEC(1) in Figure I-3 is the average net rate of return, $f_1(X_1)$, now labelled ROCE(1) – return on capital employed: this slopes downwards at half the gradient of the MEC – that is, the gradient is (negative) 0.00124375. An investment of $6.0m will earn a net return of 10.05375% (= 100*(0.108 – 6.0*0.00124375)). This grosses up to 13.964%, the same as for Figure I-2. These rates of return apply in every year, and the gross profit figure ($13.964m) appears at the head of the P&L account in the 6% growth case in Table I-1.

What is more, a similar diagram may be constructed in relation to the other growth rates shown in Table I-1, and the same rates of return will apply. The net rate of return – ROCE = 10.05375% – is in fact determined (in the 'straight line' case only) by the values taken by the intercept and the WACC: thus, the net return will equal the simple average of these two numbers (= 100*((0.108 + 0.093075)/2)). The common operating profit figure – $13.964m again – is evident at the head of each of the P&L accounts. Table I-1 includes the gradients of the MECs in each case such that capital employed will have reached the common value of $100.0m at the end of Period Zero: this must happen for each corporation at some point in its story, but it is not necessary to think that each reaches this milestone in the identical accounting period. In the case of 1% growth, for example, the gradient of the MEC is listed at

0.01493, and the optimal level of the Period One capital expenditure will then be $1m, so that capital employed at the end of Period Zero will again be $100.0m (= 1.0/0.01).

II. MAXIMISING MARKET VALUE ADDED

Integrating the Building Blocks

The previous section has introduced the four components required to build a model of the growing corporation – the WACC, the MEC (and associated ROCE (average rate of return) function, $f_1(X_1)$), the discounted cash flow model (even though no cash dividends may be paid), and finally the mechanism whereby the period by period shift in the MECs drives its growth. These will be incorporated into a single equation, so that the decision rules used in the zero growth case above may be proven and generalised. These rules are obtained by applying the differential calculus, and are embodied in two further equations, which will be analysed in the following sections of the study.

MVA

As discussed in the previous section, the corporation's market value added (M_0) is the difference between its enterprise value (V_0) and the book cost of its capital employed (A_0):

$$M_0 = V_0 - A_0 \qquad \text{(I-1)}$$

where the zero subscripts refer to the end of Period Zero – and thus the beginning of Period One also. The corporation is assumed by this point in time to have made an investment of X_0 in fixed assets, and to have distributed its residual net free cash flow, D_0, through a stock repurchase exercise. (Note that accounting and stock market magnitudes will be labelled with upper case letters, with constants and accounting and market ratios generally indicated by the use of the lower case.) The same applies to the corporation's capital employed, also: depreciation charged at the end of Period Zero is exactly the amount expended in Period Zero on maintaining the productive capacity of its existing assets. These last entries take place further up the profit and loss account than shown in Table I-1.

Enterprise value is the market capitalisation of the corporation's equity (Q_0) and debt (B_0) combined:

$$V_0 = Q_0 + B_0 \qquad \text{(I-2)}$$

and, by assuming an absence of unanticipated interest rate movements, it may be taken that the corporation's outstanding debt will trade at face value. If the corporation can issue its debt at a coupon of 9%, the resulting cash flow – interest and redemption payments – will be discounted at 9%. The MVA then becomes wholly attributable to the holders of its stock:

$$M_0 = Q_0 - (A_0 - B_0) \qquad \text{(I-3)}$$

where the bracket represents the value of the balance sheet equity, accumulated as profits have been retained over the years.

Gearing Ratio

Defining d as the gearing ratio, the ratio of debt to enterprise value, the following apply immediately:

$$B_0 = dQ_0/(1-d) \quad \text{(I-4)}$$

and:

$$V_0 = Q_0/(1-d) \quad \text{(I-5)}$$

For instance, if the debt ratio is 0.25 (25%) – as in Figure I-1 – that is:

$$B_0 = dV_0 = 0.25V_0 \quad \text{(I-6)}$$

then the market value of the debt will be one-third that of the equity, and the enterprise value will be one and one-third times the equity.

Capital Employed

Defining X_1 as the corporation's optimal level of capital expenditure at the end of Period One, it has already been argued in relation to Figure I-3 that the book value of the assets at the end of Period Zero will amount to A_0 where:

$$A_0 = X_1/g \quad \text{(I-7)}$$

(Note that the use of the asterisk to denote optimal or solution values of the variables will be abandoned where no confusion arises.) The parameter g is the periodic shift in the MEC function, and this applies also to the rate of return function, $f_1(X_1)$: in the following period, $f_2(X_2)$ will have a lesser gradient than its predecessor. A_0 is the accumulated book cost of all previous amounts invested by the corporation, and may thus be identified with capital employed at the end of Period Zero.

Dividend Discount Model

When, at the end of each period, there is a cash surplus in the corporation's bank account, it engages in a stock repurchase programme. The dividend discount model no longer applies at the level of the single share, but at the corporate level, of course, there is a cash disbursement equal to the residual net free cash flow – the available surplus after payment of the period's corporation tax, and after investing the optimal amount in capital expenditure.

Assuming these disbursements increase from year to year at the growth rate, g, the present value amounts to the equity market capitalisation:

$$Q_0 = D_0 \sum [(1+g)^t/(1+r(d))^t] \qquad \text{(I-8)}$$

where the summation extends over the range t = 1 through infinity, and D_0 is the amount of the repurchase programme at the end of Period Zero. The function, r(d), is the discount rate: it is the upward-sloping ECC shown previously in Figure I-1. Provided that the discount rate is greater than the growth rate, the infinite sum converges to give:

$$Q_0 = D_0(1+g)/(r(d) - g) = D_1/(r(d) - g) \qquad \text{(I-9)}$$

that is, equity market capitalisation will be the prospective disbursement at the end of Period One divided by the excess of the ECC over the growth rate. In order to value the corporation, this equation is more conveniently written:

$$Q_0(r(d) - g) = D_1 \qquad \text{(I-10)}$$

where the bracket post-multiplying the equity market capitalisation is the residual net free cash flow yield – or 'disbursement yield', as it is labelled in Table I-1. In view of the decision to effect the annual distribution through a stock repurchase plan, it would now be misleading to talk in terms of the more common 'dividend yield'.

Profit and Loss Account: Period One

By the end of Period One, the corporation will have accumulated residual net cash for disbursement of:

$$D_1 = A_0 f_1(X_1) - B_0 n(d) - X_1(1 - B_0/A_0) \qquad \text{(I-11)}$$

The first term on the right-hand side of this equation is the NOPAT, net operating profit after tax. It is the product of the capital employed at the end of Period Zero and the return on capital employed – ROCE – (after corporation tax, of course): Figure I-3 above indicates that the corporation invests at the same rate of return each period, and that that rate of return may be determined by solving for the optimal level of capital expenditure at the end of Period One. It has already been established – Equation I-7 above – that X_1/g may be used to substitute for A_0.

The second term is the net interest to be paid at the end of Period One: it is the product of the debt outstanding at the end of Period Zero and the rate of interest, n(d) (again net of corporation tax): this relationship was labelled INTRTE in Figure I-1. The interest rate is of course constant from period to period, as deduced from the figure, and its value will be solved for by examining the decisions taken during Period One. Here B_0 in Equation I-11 above may be replaced by $dQ_0/(1-d)$ as shown in Equation I-4. For later reference, the first two terms on the right of Equation I-11 amount to the corporation's earnings: this is labelled as net profit in Table I-1.

The final term on the right is the amount retained by the corporation to finance the equity component of the Period One capital expenditure: the expression in the bracket is the equity financing proportion in terms of book values. Again Equation I-4 is used to solve for B_0, and Equation I-7 is used to eliminate the inverse of A_0 $(= g/X_1)$.

Equation I-11 thus represents a brief statement of the P&L account for Period One: in Table I-1 above, the P&L accounts are extended upwards to begin with operating profit, so that gross interest payments and corporation tax payments may be made explicit, as in conventional financial accounts. The indicated substitutions result in:

$$(r(d)-g)Q_0 = X_1 f_1(X_1)/g - n(d)dQ_0/(1-d) - X_1 + gdQ_0/(1-d) \qquad \text{(I-12)}$$

where, on the left-hand side, the revision makes use of Equation I-10, and the last term is now explicitly the amount the corporation will add to its outstanding borrowings at the end of Period One – that is, the growth rate times the initial period debt.

Enterprise Value

Equation I-5 may be used to reformulate this expression in terms of V_0:

$$(r(d)-g)(1-d)V_0 = (X_1 f_1(X_1)/g - X_1) + (dg - dn(d))V_0 \qquad \text{(I-13)}$$

which may then be solved for the enterprise value to give:

$$V_0 = (X_1 f_1(X_1)/g - X_1)/z \qquad \text{(I-14)}$$

where:

$$z = (1-d)r(d) - g + gd - dg + dn(d) = (1-d)r(d) + dn(d) - g$$
$$= WACC - g \qquad \text{(I-15)}$$

In the previous section, the WACC was defined as the market capitalisation weighted sum of the equity cost of capital and of the interest rate:

$$WACC = (1-d)r(d) + dn(d) \qquad \text{(I-16)}$$

so that Equation I-14 translates to:

$$V_0 = A_0(\mathrm{ROCE} - g)/(\mathrm{WACC} - g) \qquad \text{(I-17)}$$

which becomes the basic valuation formula. The substitutions in the numerator again make use of Equation I-7, including $X_1 = gA_0$. It was this formula that was used to value the zero growth corporation in Section I. In that special case, capital employed worked out at $100.0m, and g took the value zero. Enterprise value in this case, $108.018m, was simply capital employed inflated by the ratio of the ROCE (10.05375%) to the WACC (9.3075%).

The final step in the derivation is to substitute for V_0 and A_0 into Equation I-1 above for the MVA to give:

$$M_0 = (X_1 f_1(X_1)/g - X_1)/z - X_1/g \qquad \text{(I-18)}$$

Equation I-18 expresses the MVA in terms of the decision variables, the gearing ratio and the Period One capital expenditure, and also the shift factor in the MEC, g.

Solving for the Decision Variables

First order conditions for the maximisation of MVA are obtained by differentiating M_0 with respect to d and X_1, and setting the resulting partial derivatives equal to zero. This process results in:

$$z'(d)(X_1 - X_1 f_1(X_1)/g)/z^2 = 0 \qquad \text{(I-19)}$$

and:

$$((X_1 f_1'(X_1) + f_1(X_1))/g - 1)/z - 1/g = 0 \qquad \text{(I-20)}$$

where the said derivatives appear on the left-hand side in each case. $z'(d)$ indicates the derivative of z with respect to d, and $f_1'(X_1)$ indicates the derivative of $f_1(X_1)$ with respect to X_1. These two equations will be explained and solved, by reference to Figures I-1 and I-3, in the following two sections of the study.

Second order conditions for a maximum will also be discussed in Part II of the study, but in the meantime it may be noted that these will be satisfied so long as n(d) and r(d) slope upwards against the gearing ratio, and $f_1(X_1)$ slopes downwards against the amount of capital expenditure. It will further be necessary that the gradient of n(d) exceeds that of r(d): that is, bond investors are assumed to be more risk averse than equity investors.

It is pertinent to remark that there is nowhere in the derivation the suggestion that either the payout ratio or the retention ratio is to be considered as a decision variable of the model. The development of the system over time is fully determined by the gearing ratio and the Period One capital expenditure outlay. That this provides a general solution to the problem of valuing the corporation follows from the fact that any two successive years in its life could be labelled Period Zero and Period One. In the simulations in Section V, it will be helpful, in making comparisons between the different growth rates, to choose Period One to coincide with the year in which the corporation's capital employed stands at \$100.0m in each case.

As a footnote to this section, it is appropriate to note that it is possible to rework the derivation of Equations I-19 and I-20 to incorporate personal taxes, where these are not so easily avoided as is the case under the present UK system. It is intended to pursue this topic in a subsequent study.

III. THE OPTIMAL GEARING RATIO

The Trade-Off Theory of Capital Structure

To some extent, the present study represents an exercise in reverse engineering the theory put forward by Brealey, Myers and Allen (2006). While their model results in a significantly different conclusion from that arrived at by Modigliani and Miller (1959), it is clear from the literature that it has by no means eclipsed the earlier theory. Without reference to the Brealey, Myers and Allen thesis, Ross (1989) endorses the approach they adopt, though with some reluctance it seems: 'This is logically possible, but at the expense of reducing the study of corporate finance to the trade-off between the tax advantages of debt and the costs of bankruptcy.' Fortunately, a somewhat more encouraging reading is offered by Koller, Goedhart and Wessels (2005). Their reservations relate more to the practical aspects of the theory, and in particular to the insensitivity of enterprise value to the level of gearing. Discussion of their attitude will be reserved for Part II of the study, by which stage the analysis will have laid bare the conditions for the existence of an optimal gearing ratio.

Brealey, Myers and Allen (2006) argue that the theory proposed by Modigliani and Miller (1959) fails to take account of the potential costs of financial distress, and postulate that the corporation will in fact face an upward-sloping interest rate function: the greater the risk of default, the greater the interest rate it will be charged on its borrowings. Figure I-1 in Section I is drawn to demonstrate this effect, and in the zero growth case examined therein, resulted in an interest rate of 9% gross. The interest rate function shown implies that, by increasing its gearing ratio to 30%, say, the corporation would be forced to pay interest at the rate of 9.839% gross: undated stock issued with a coupon of 9% could only be sold at 91.474% of face value. This discount represents an up-front (additional) insurance premium charged by the corporate bond market to reflect the greater risk associated with the more highly leveraged situation. The greater the degree of risk aversion on the part of the suppliers of debt finance, the greater will be the gradient on the interest rate function, and the more punishing the premium. (Of course, it has to be remembered too that the straight line picture is likely to be a gross over-simplification of the way the bond market prices the risks associated with fixed interest investment.)

The upward slope in the ECC similarly reflects risk aversion on the part of the equity market. The more debt the corporation takes on, the higher will be the volatility of the residual profits left over for the stockholders. The higher intercept shown for the ECC reflects the higher risk premium applicable to equity investment.

Equation I-19 Simplified

As noted, this section aims to simplify Equation I-19 and to relate it to Figure I-1 in Section I. Firstly, it may be noted that multiplying through the equation by z^2 eliminates this term altogether. Secondly, the expression in brackets is merely a scalar (that is, it does not entail the appearance of d, the debt ratio), and may similarly be multiplied out: the value of the scalar will affect only the shape of the curve, not the solution value of d. Thus the equation to be solved may be written:

$$z'(d) = 0 \qquad \text{(I-21)}$$

whence, after differentiating Equation I-15 above with respect to the gearing ratio, Equation I-21 may be seen to be equivalent to:

$$(1-d)r'(d) - r(d) + dn'(d) + n(d) = 0 \qquad \text{(I-22)}$$

which means that the slope of the WACC has to be zero (the rate of growth, present in Equation I-15, does not appear in the differential). Subject to the second order conditions being satisfied, this means that the gearing ratio should be selected so as to determine that the WACC will be at its minimum. This equation may be rearranged to give:

$$n(d) + dn'(d) = r(d) + dr'(d) - r'(d) \qquad \text{(I-23)}$$

Now on the left-hand side of the equation, the marginal cost of debt (MCD) has emerged: on the right is the marginal cost of equity (MCE), except that the MCE has to be shifted down along its length by the slope of the ECC: in general, this size of this reduction will depend on the value of d.

Marginal Cost of Debt (MCD)

In Figure I-1 above, the interest rate function (adjusted for the tax shield, of course) is shown as a straight line, with equation:

$$n(d) = 0.0346 + 0.1208d \qquad \text{(I-24)}$$

Differentiating this equation with respect to d gives the constant, 0.1208. The equation shows how the corporation will be charged interest at a higher rate, the higher its gearing ratio, and therefore its risk of bankruptcy. The debt ratio is measured in terms of market values, as the market is assumed to concern itself with the economic value of the corporation, rather than with book values. The marginal cost of debt may then be seen to be:

$$MCD = n(d) + dn'(d) = (0.0346 + 0.1208d) + 0.1208d$$

$$= 0.0346 + 0.2416d \qquad \text{(I-25)}$$

so that the MCD has the same intercept as the interest rate function, but twice the gradient – given the straight line assumption. These relationships are shown as INTRTE and MCD in Figure I-1.

Adjusted Marginal Cost of Equity (MCE-)

The equity cost of capital (ECC) is also represented as a straight line in the figure in order to keep the analysis as simple as possible (this will generally be a suitable approximation, given that only a limited segment of the relationship will be material to the simulation). This time the equation is:

$$r(d) = 0.1 + 0.01d \qquad \text{(I-26)}$$

which again slopes upwards with the gearing ratio. The first derivative of this equation is 0.01, so that the right-hand side of Equation I-23 becomes:

$$MCE- = r(d) + dr'(d) - r'(d) = (0.1 + 0.01d) + 0.01d - 0.01$$

$$= 0.09 + 0.02d \qquad \text{(I-27)}$$

where MCE- is used to denote the marginal cost of equity as adjusted downwards by the derivative (in this case, a constant (= 0.01)) along its length. These relationships are shown as ECC and MCE- in Figure I-1.

The Optimal Gearing Ratio

These marginal linear relationships intersect as indicated by Equation I-23:

$$0.0346 + 0.2416d = 0.09 + 0.02d \qquad \text{(I-28)}$$

to give the optimal gearing ratio as 0.25 (= 0.0554/0.2216), or 25%. This value of d (again the asterisk has been dropped) may be plugged back into the relationships to determine the corresponding values of the interest rate and the equity cost of capital:

$$INTRTE = n(d) = 0.0346 + 0.1208 * 0.25 = 0.0648 \qquad \text{(I-29)}$$

and:

$$ECC = r(d) = 0.1 + 0.01 * 0.25 = 0.1025 \qquad \text{(I-30)}$$

that is, 6.48% and 10.25% respectively: these were the values used earlier in the zero growth simulation. The interest rate grosses up to 9% in computing the P&L accounts in Table I-1.

Weighted Average Cost of Capital (WACC)

The minimal value taken by the WACC is then:

$$\text{WACC} = (1-\text{d})\text{r}(\text{d}) + \text{dn}(\text{d}) = 0.75 * 0.1025 + 0.25 * 0.0648$$

$$= 0.093075 \qquad \text{(I-31)}$$

or 9.3075%. This value may be observed in Figures I-1 through I-3. It is the minimum value of the curve for the WACC in Figure I-1, and this solution value then appears as a constant in the subsequent two figures.

It may be noted that the marginal relationships intersect at a slightly higher value:

$$\text{MCD} = 0.0346 + 0.2416 * 0.25 = 0.095$$

$$= 0.09 + 0.02 * 0.25 = \text{MCE -} \qquad \text{(I-32)}$$

which indicates that, at the optimal gearing ratio, an extra dollar of debt will cost the corporation 9.5c post tax per annum in interest: the interest rate will be only fractionally higher on the marginal dollar, but the higher rate will apply to the whole of the outstanding debt burden. This figure plays no further part in the analysis.

By eliminating the Period One capital expenditure from Equation I-19, it has been shown that the gearing ratio and the WACC are to be treated as constants, in the sense that they may be considered independent of time and the level of capital expenditure. As g failed to survive the differentiation of z, they are also independent of the rate at which the corporation is growing.

Conditions for the Existence of the Optimal Gearing Ratio

As noted earlier, a case can be made out for the upward-sloping INTRTE (n(d)) and ECC (r(d)) functions shown in Figure I-1 above, and this then results in the upward slopes of the MCD and MCE- relationships as explained above. Beyond that, it has been necessary to embrace the requirement that bond investors will be more risk averse than equity investors: this is demonstrated by the fact that the gradient of INTRTE is greater than that of the ECC. The final condition for the optimum to exist is that the MCD and the MCE- should intersect at a value of the gearing ratio less than unity – that is, $d<1$. In due course, it may be shown that a combination of a high optimal debt ratio (low bondholder risk aversion) and a high growth rate will result in a Modigliani–Miller type of solution, with the corporation’s capital expenditure in each period financed entirely by adding to its outstanding debt, and the whole of its earnings distributed to the stockholders – thus implying a 100% payout ratio.

IV. THE OPTIMAL LEVEL OF PERIOD ONE CAPITAL EXPENDITURE

Equation I-20 Simplified

This section aims to simplify Equation I-20 and to relate it to Figure I-3 above. The equation may be rearranged to give:

$$f_1(X_1) + X_1 f_1'(X_1) = z + g \qquad \text{(I-33)}$$

which, on substituting for z from Equation 15, may be written:

$$f_1(X_1) + X_1 f_1'(X_1) = (1-d)r(d) + dn(d) \qquad \text{(I-34)}$$

The expression on the left will be shown, using the relationship in Figure I-3, to be the marginal efficiency of capital expenditure (MEC). The expression on the right is the familiar formula for the WACC (Equation I-31), which has already been evaluated as 0.093075 in the previous section. It is then possible to solve the equation for X_1.

Marginal Efficiency of Capital Expenditure (MEC)

The equation for ROCE(1) in Figure I-3 above is given by:

$$f_1(X_1) = 0.108 - 0.00124375X_1 \qquad \text{(I-35)}$$

The derivative of this equation with respect to X_1 is the constant (negative) 0.00124375, so that the left-hand side of Equation I-34 may be rewritten:

$$f_1(X_1) + X_1 f_1'(X_1) = (0.108 - 0.00124375X_1) - 0.00124375X_1 \qquad \text{(I-36)}$$

or, alternatively, substituting for the WACC:

$$\text{MEC}(1) = 0.108 - 0.0024875X_1 = 0.093075 \qquad \text{(I-37)}$$

wherein the marginal relationship has the same intercept as Equation I-35, and the gradient is doubled. At the second equality, Equation I-37 may be solved to give:

$$X_1 = 6.0 \qquad \text{(I-38)}$$

that is, the corporation's optimal level of Period One capital expenditure will be \$6.0m. Substituting this value for X_1 in Equation I-35, it may be calculated that the ROCE works out at 10.05375% as before.

Capital Employed

If then, as shown in Figure I-3 above, the MEC curves are swinging to the right over time at the rate of 6% per annum, the corporation's capital employed at the end of Period Zero, A_0, will stand at \$100m (= 6.0/0.06): this was argued in relation to Equation I-7 previously. The shift in the MEC may be modelled as a steady reduction of the slopes of the lines. For instance, in Period Two, the corporation is faced with the rate of return function:

$$f_2(X_2) = 0.108 - 0.00124375X_2/(1+g) = 0.108 - 0.00117335X_2 \quad \text{(I-39)}$$

for $g = 0.06$. The corresponding MEC is then:

$$MEC(2) = 0.108 - 2.0 * 0.00117335X_2 \quad \text{(I-40)}$$

which, when set equal to the WACC (= 0.093075) returns:

$$X_2 = 6.36 \quad \text{(I-41)}$$

that is, $6.36m, the corporation's optimal level of capital expenditure in Period Two. It is a matter of mental arithmetic to verify that:

$$X_2 = (1+g)X_1 \quad \text{(I-42)}$$

and that:

$$A_1 = X_2/g = 6.36/0.06 = 106.00 = (1+g)A_0 \quad \text{(I-43)}$$

where A_1, of course, stands for the capital employed at the end of Period One. The MEC curves are moving over time in such a way as to lead the optimising corporation to grow its capital employed by 6% per annum.

In fact, the model is constructed in such a way that the labelling of the periods is completely arbitrary, so that the solution is completely general: the model may be solved for any point in the life of the corporation. It is then a matter of convenience to take the year referred to as 'Period One' as that point in time at the commencement of which capital employed stands at $100.0m. This permits more penetrating comparisons to be made between corporations with different growth rates, even though the comparisons may be being made between companies reaching this milestone in different years.

V. CENTRAL SIMULATION

Layout: Table I.1

Table I.1 presents P&L accounts, balance sheets, and other accounting and stock market statistics for ten corporations growing at rates between zero and 9% per annum. Each of these corporations is moving along a steady-state growth path, in the sense that each of its accounting and stock market magnitudes grows at the rate shown at the head of the column, while each of its accounting and stock market ratios remains constant over time. It has already been established in Section III (Equation I-30) that the equity of each of the ten corporations will be priced in the market to return 10.25%: this is the ECC, given the optimal level of the gearing ratio. By rearranging Equation I-9, the dividend discount model then shows that the sum of the growth rate and the yield should be equal to the ECC: except that, as argued earlier, it is now necessary to think in terms of the 'disbursement discount model' and the 'disbursement yield' to take account of the tax advantages of stock repurchases over the payment of pro rata cash dividends. As the growth rate increases, so the disbursement yield will decline point for point.

The Case of 6% Growth Concluded

The study continues to focus on the case of 6% growth, which is therefore highlighted in Table I-1: the solution process is then readily extended to other growth rates, including the zero case already detailed in Section I.

The first step is to solve for V_0, the enterprise value at the beginning of Period One, using Equation I-17 above:

$$V_0 = 100.0(0.1005375 - 0.06)/(0.093075 - 0.06) = 122.562 \qquad \text{(I-44)}$$

where the parameters are now familiar as, respectively, capital employed, the ROCE, the growth rate, and the WACC (and the growth rate again). The enterprise value then splits three-to-one between equity market capitalisation and debt:

$$Q_0 = 91.922 \qquad \text{(I-45)}$$

and:

$$B_0 = 30.641 \qquad \text{(I-46)}$$

with, finally, the optimand emerging at a value of:

$$M_0 = V_0 - A_0 = 22.562 \qquad \text{(I-47)}$$

The market value added is the net present value of the excess profits to be generated by the corporation over an infinite future life, both on its existing capital employed and on all the investments it will make in later years. This latter amount is usually referred to as the PVGO, the present value of the corporation's growth opportunities.

Establishing a value for B_0 (Equation I-46) permits the P&L account to be completed. The 6% column begins with operating profit of $13.964m, the grossed-up value of the ROCE calculated from Equation I-35: that is, $10.05375/(1.0 – 0.28)m. The grossed-up interest rate is 9% (= 6.48/(1.0 – 0.28)) as before, so that interest on $30.641m amounts to $2.758m, leaving profit before tax of $11.206m. Corporation tax at 28% on this amount absorbs $3.138m, resulting in net profit (earnings) of $8.068m. The balance sheet at the top of the page indicates that some 69.359% (this is the complement of the debt figure computed earlier) of the planned $6.0m capital expenditure should be financed by retentions: this amounts to $4.162m, leaving the residual net free cash flow of $3.907m. This is then the amount available for payout in the form of a stock repurchase plan. That the accounts balance is demonstrated by the fact that this disbursement represents 4.25% (labelled as the 'disbursement yield') of the equity market capitalisation as calculated above. The somewhat unusual form of the P&L account has been alluded to earlier, and will be discussed below, but it is not normal accounting practice to provide for the payout as if it were a commitment in the way that a cash dividend is.

Under accounting ratios, the first item is the return on capital employed (ROCE) at the NOPAT level – 10.05375%: this may be alternatively be equated with the operating profit ($13.964m) less the corporation tax payable ($3.138m) and less the value of the tax shield – 28% of the interest paid ($0.772m). The return on equity is obtained as the ratio of net profit to balance sheet equity. The payout ratio is the ratio of the repurchase plan to the net profit, and interest cover is the ratio of operating profit to the gross amount of the interest paid.

Among the stock market statistics, the P–E ratio is equity market capitalisation divided by net profit, while the PEG ratio is the P–E ratio divided by the growth rate.

The overall tax rate is computed on the assumption that, given the planned stock repurchase, the corporation can ignore income and capital gains tax – as, indeed, was assumed to be the case for the calculation of the WACC. The tax rate is obtained as the ratio of the corporation tax paid to the total returns received by the stakeholders, including the tax authorities: this is then the sum of the tax paid (this is equal to the numerator – $3.138m), the gross interest received by the bondholders ($2.758m) and $9.422m attributable to the stockholders. This last amount represents 10.25% (the ECC) of the initial period equity market capitalisation ($91.922m). This gives an overall tax rate of 20.484%, a considerable saving on the 28% rate of corporation tax officially levied in the UK. The return to the stockholders – $9.422m as noted – includes the repurchase plan – $3.907m – and capital appreciation of $5.515m: this last figure represents 6% on the beginning period equity market capitalisation. By contrast, the share price will have risen by 10.25%, given that the corporation will have fewer shares in issue after the repurchase plan has been implemented.

The abatement of the tax rate reflects not only the tax shield effect – the figure of $0.772m noted above – but also the tax free element of the capital appreciation accruing to stockholders. Insofar as the gain of $5.515m is backed by an increase in the balance sheet value of the equity – equal to retentions of $4.162m – it will have suffered corporation tax, of course. But this leaves a difference of $1.353m, which may be interpreted as the increase in the MVA – 6% of $22.562m. This amount accrues to stockholders entirely free of tax, assuming as always that capital gains tax may be ignored.

Finally, at the foot of the 6% column, the parameters of the MEC(1) function are repeated: the intercept of 0.108 and the gradient of (negative) 0.0024875, as described earlier in relation to Figure I-3 and Equation I-37.

The Stock Repurchase Plan

Again, it is appropriate to consider the impact of the disbursement of the residual net free cash flow through the repurchase programme. As noted previously, during Period One the equity market value of the corporation should rise by 10.25%, some $9.422m, to $101.344m. Capital appreciation of 6% – $5.515m – is directly attributable to the underlying growth of the corporation, with the balance – $3.907m – reflecting the build-up of distributable cash (that is, after the implementation of the capital expenditure programme). If the corporation has 100m shares in issue at the end of Period Zero, now priced at $1.0134 (cum) each, the cash will repurchase 3.855m (= 3.907/1.01344) shares, leaving 96.145m outstanding. Period Two net profit will be 6% higher than year one at $8.552m, so that prospective earnings per share for Period Two will then be 8.895c (= 8.552/96.145), 10.25% greater than earnings per share of 8.068c in Period One, as shown in Table I-1. As with the case of zero growth, earnings per share appear to grow at a rate equal to the ECC, even though the corporation is actually growing at the underlying rate of 6% per annum, as shown by the fact that this is the growth rate of its net profit and equity market capitalisation. Compared with a conventional cash dividend, the repurchase alternative is the equivalent of ongoing stockholders using their ('cash') dividend to increase their holdings in the corporation: by comparison, at the end of the buy-back process, they will hold fewer shares trading at a correspondingly higher price.

The repurchase programme results in the price per share and the earnings per share increasing at a rate equal to the ECC rather than the underlying growth rate. Again the adjustment factor to be applied to the historic (Period One) earnings is the ratio of the number of old shares in issue to the current number: this works out at 1.0401 (= 100.0/96.145). Earnings per share of 8.895c in Period Two will then work out 6% greater than the adjusted figure of 8.391c (= 1.0401*8.068) in Period One.

In practice, the situation tends to be more complicated than this, with many corporations implementing repurchases, while at the same time continuing to pay cash dividends also. In addition, such buy-backs may often be financed by funds released by selling surplus assets or by issuing additional debt. A study by Secker and Swing (2007) shows that 58% of UK companies implemented repurchase programmes in 2006, amounting to some £46b. This figure compares with £62b. paid out in cash dividends. At the same time, UK corporations raised £23b. in new equity. There is no way of knowing to what extent repurchases were financed out of 2006 earnings, and to what extent by capital transactions: for instance, mining companies certainly showed a preference for reflecting the high level of metals prices by boosting buy-backs, rather than taking the risk of increasing cash dividends beyond sustainable levels.

Other Growing Corporations

The accounts for the 6% growth case provide a paradigm for computing the other cases in the table. The accounts have been normalised to show capital employed of $100.0m in each case by choosing the slope of the MEC function to give the appropriate optimal level of capital expenditure at the end of Period One. In the case of 1% growth for instance, the marginal efficiency of capital expenditure is given by:

$$\mathrm{MEC}(1) = 0.108 - 0.01493\mathrm{X}_1 \qquad \text{(I-48)}$$

These parameters are quoted at the foot of the 1% column in Table 1. Setting the MEC equal to the WACC – 9.3075% (0.093075) as before – leads to an optimal outlay of $1.0m at the end of Period One. Applying the perpetuity formula as usual, capital employed at the end of Period Zero will be computed as $100.0m (= 1.0/0.01). As before, the gradient of the ROCE will be half that of the MEC to give:

$$\mathrm{ROCE}(1) = 0.108 - 0.5 * 0.01493 * 1.0 = 0.1005375 \qquad \text{(I-49)}$$

so that the rates of return, both gross and net, on this investment will be the same as for the 6% case. The balance sheet and the P&L account then follow in the same way.

Comparisons between the cases in Table I-1 will be discussed in Part II of this study. In the meantime, it is relevant to note that the corporations are priced on a no-arbitrage basis: each offers investors the same rate of return – the ECC, 10.25% – which in this case is received in the form of an annual increase in the value of the investor's holding. The study thus has no contribution to make to the controversy as to whether higher yielding stocks tend to generate higher overall rates of return than their lower yielding counterparts. This possibility was examined by Litzenberger and Ramaswamy (1982), who came to the conclusion that stocks with higher dividend yields tend to deliver higher total returns: their finding was in conflict with an earlier study by Black and Scholes (1974).

Stasis: The Case of Zero Growth

The earlier discussion (Section I) envisaged zero growth as the result of a once-off optimisation exercise: this had the advantage of introducing the concept of the marginal efficiency of capital employed. Where stasis does exist in the corporate world, however – and it would be unusual for there to be no trend in either direction in the earnings of a company – it is in fact more likely to occur where an industry has exhausted its growth opportunities. The importance of the zero growth column is to provide a starting point for the computation of the payout ratio. The accounts for Period One are identical to those of Period Zero, and every other period in this case, and the payout ratio works out at 100%.

As noted previously, reported earnings will show a 10.25% rate of increase year to year as the number of shares in issue declines over time in accordance with the repurchase policy. Indeed, without adjustment, each of the companies depicted in Table I-1 will manifest earnings per share growth equal to the ECC, in spite of their different rates of progress in economic terms.

The Case of 10% Growth

An inspection of the valuation formula, Equation I-17, shows that the denominator vanishes when the growth rate reaches the level of the WACC – 9.3075% in this case: the value of the corporation becomes highly unstable as this critical level is approached, as may already be seen in the heady multiple (57.085) put on the earnings of the corporation growing at 9%. It is, in fact, still possible to find a meaningful solution for the case of 10% growth, as long as the ECC is greater than this: this is shown by Equation I-9. The solution may be obtained by introducing the constraint that balance sheet equity should be non-negative – that is, $B_0 \leq 100.0$ for a corporation with capital employed of $100.0m. This is anyway a legal requirement in the UK and the US, and indeed most other jurisdictions: a corporation may borrow and pay out the proceeds to its stockholders, but only if there is adequate cover available in its accumulated stockholder reserves (and the bondholders agree, of course). It is intended to leave the investigation of outcomes of this type to a subsequent study, but it may be noted that the constraint becomes binding in the case of 10% growth. As a result, the optimal debt ratio is lower than the 25% level used in the Table I-1 simulations. Both the interest rate and the ECC are lower, but the WACC is greater. This reflects a significant escalation in the equity market capitalisation consequent upon the corporation enjoying a higher ROCE and the faster growth rate.

This case is of interest because it generates a solution of the type envisaged by Modigliani and Miller (1959). The corporation finances its optimal capital expenditure each period only by adding to its outstanding debt: the whole of its net profit will be available for distribution to its stockholders. The optimal payout ratio will be 100% – the same as for the corporation in stasis. In the next section it will be possible to identify the critical growth rate such that full distribution becomes appropriate. In the meantime, reference to Figure I-1 will indicate the part played by risk aversion on the part of the suppliers of debt finance in pushing up the payout ratio in this manner. A lesser gradient for the interest rate function would result in a higher optimal gearing ratio, which in turn could imply that the corporation growing at 9% would find it appropriate to adopt a 100% payout ratio. In general, the lower the gradient of the interest rate function, the lower will be the threshold rate of growth such that the capital expenditure will be financed exclusively with debt. If interest rates are low in this sense, the corporation should gear up as far as the markets will allow, as indicated by the Modigliani–Miller model, or at least as far as legal framework will allow.

Residual Net Free Cash Flow

This subsection aims to provide an explanation of the last entries in the P&L accounts shown in Table I.1 above, which appear to defy standard accounting practice. In the normal way, the P&L account would show the corporation's proposed cash dividend for the period, with retentions taken as the residual. Treating retentions as the prior commitment conforms with the Modigliani–Miller approach whereby the corporation gives priority to making its optimal capital investment each period. Thereafter, as the corporation is assumed, for tax reasons, to distribute its dividend pot through a stock repurchase programme, it seems inappropriate to label this as a proposed dividend. Thus it has been labelled as the 'payout'. The usual practice would be to treat the whole of the net profits as retentions, and then, in the following year, to reveal in the cash flow statement how much had been deployed in relation to any stock repurchase programme previously approved by the stockholders. In practice, the stock repurchase programme might also encompass the utilisation of funds released through corporate activity – for instance, as a result of the disposal of assets, or the substitution of debt finance for equity. Certainly, corporations prefer to use the repurchase process, since it gives them a lot more flexibility, and in particular, defuses the imperative always to maintain – at the very least – the cash value of the cash dividend. Stonham (2002) notes that many managements resent the upwards-only dividend treadmill, and use the repurchase process as a means of temporarily boosting their payouts – for instance, after a particularly good run of results.

Table I-2 offers an explicit view of the corporation's cash flows for the case of 6% growth, under the simplification that all transactions relating to Period One actually take place in Period One, including the buy-back. Under sources, the operating profit is taken directly from the data in Table I-1. It has been possible to ignore depreciation up to this point, but now it is assumed that the corporation's assets have a life of ten years, and that it therefore writes off one tenth of its $100.0m of capital employed in Period One, and simultaneously expends $10m on maintaining the earning capacity of its assets. (The assumption that there is no difference between the corporation's depreciation charge and its associated capital allowances for tax purposes was noted earlier.) The other source of funds is the additional borrowing the corporation will undertake at the end of Period One: this is simply taken at the rate of 6% – the growth rate – on the initial debt position ($30.641m). Sources thus total $25.802m, as shown in the table.

Table I-2 Cash Flow Statement for Period One (Growth = 6% PA)

		$m	$m
Sources			
	Operating profit	13.964	
	Depreciation	10.000	
	Borrowings	1.838	
			25.802
Uses			
	Interest paid	2.758	
	Corporation tax	3.138	
	Capital expenditure	16.000	
	Stock repurchase	3.907	
	Total		25.802
Change in net working capital			0.000

The first two items on the list of uses, interest and tax, are drawn directly from the P&L account. The $16.0m capital spend includes the $10.0m replacement programme already discussed, and the $6.0m expansion programme. The latter amount may be deemed to be financed by the additional borrowings shown under sources, and by retained profit – the equity component – as shown in the P&L account. Finally, the stock repurchase plan for Period One completes the uses list, even though, as noted above, in practice corporations would normally include such outlays a year in arrears, rather than make specific provision in the P&L and implement the repurchase within the same accounting period. This treatment avoids complicating the model unnecessarily.

As set out, the flow of funds statement in Table I-2 shows no change in working capital. This simplification means that it is assumed that debtors, creditors and stocks – and possibly a cash component as well – all grow at 6%: and that current assets are financed exactly by current liabilities, so that capital employed is then commensurate with the fixed assets that were the subject of the depreciation charge introduced above. It would not be helpful to build a more complicated cash flow statement. The objective has been to reveal the definitional problems caused by the need to acknowledge the tax situation by providing for the repurchase plan in the same way that corporations normally provide for their cash dividends. However, to avoid confusion, there is an argument for saying that corporations should indicate to stockholders how much of their current earnings they believe they are distributing.

This would also be useful information for the compilers of indices, who are otherwise in danger of publishing understated information on yields and payout ratios. Market strategists attempt from time to time to adjust dividend yields by adding in the estimated value of stock repurchases by index constituents, but this carries with it the danger of reversing the error, since their figures will then include buy-backs financed by asset disposals or by raising additional debt.

The Dividend Irrelevancy Proposition

The data in Table I-1 permit an explicit demonstration of the Dividend Irrelevancy Proposition of Modigliani and Miller (1959): this was described in the Introduction earlier. Taking, as usual, the accounts of the 6% growth company, and assuming for the moment that personal taxes do not apply, so that it can pay out a cash dividend without penalising its stockholders, the amount to be distributed would be $3.907m. This represents 4.25% of the equity market capitalisation of the corporation: adding in growth of 6% in accordance with the dividend discount formula indicates an overall rate of return of 10.25% – the ECC.

Modigliani and Miller contend that the corporation can control its dividend yield by increasing its payout by anything up to the full amount of its earnings, some $8.068m in this case. For instance, a 100% payout would increase the prospective yield on the stock to 8.777%: this is the reciprocal of the P–E ratio of course. However, in order to finance the equity portion of its $6.0m capital spending plan, it would now need to raise new equity of $4.162m. If, as in the earlier calculation, it is assumed the corporation has 100m shares in issue at the beginning of Period One, each will be priced at $1.01344 (= 1.1025*91.922/100.0) cum dividend at the end of the year, where the ECC again acts as the scaling factor. The price will fall back to 93.276c after a dividend of 8.068c per share has been paid. At this price, the corporation will need to issue 4.462m (= 4.162/0.93276) shares to raise the new equity. In Period Two, net profit will rise by 6% to $8.552m (= 8.068*1.06), but earnings per share will increase only to 8.185c (= 8.552/104.462): this represents earnings per share growth of only 1.473%, as a result of the larger number of shares now in issue. This rate of increase brings the total return earned by the stockholders back up to the 10.25% (= 1.473 + 8.777) benchmark, the sum of the increase in earnings – and thus the increase in the price of the stock also – on the one hand, and the inflated yield calculated above, on the other. This calculation again illustrates the dividend discount model at work for the case of constant growth. A prospective dividend of $$3.907m growing at 6% has the same capital value as a prospective dividend of $8.068m growing at 1.473% when discounted at 10.25%: this is the figure of $91.922m shown in the table. The value of the corporation is thus seen to be independent of the payout ratio.

The inherent circularity of the Dividend Irrelevancy Proposition is laid bare when the corporation makes its distributions by buying back its own stock in the marketplace. For then, with full distribution, it would spend $8.068m to buy stock at the 'cum dividend' price of $1.01344 per share (7.961m shares), and find itself simultaneously having to sell new shares in the amount of $4.162m (4.107m shares) at the same price. The contra-entries – purchases and sales at the same price – have no economic impact of course: the net effect is the purchase of 3.855m shares at the 'cum dividend' price. This is the same number of shares as that calculated in the subsection on the repurchase plan earlier, and the figure for the stock repurchase in Table I-2 above ($3.907m) then remains valid at the net level. In view of the robustness of this figure, it seems only logical to conclude that there is indeed an optimal payout ratio: this is 48.420% as shown under the case of 6% growth in Table I-1.

It is then tempting to make the same argument in the case of the inflated cash dividend discussed previously. Market participants will see through the offsetting cash movements, and will respond by netting off the amount of the equity financing against the higher dividend to disclose the only meaningful calculations of the corporation's disbursement and payout ratio. However, this ignores the fact that the corporation that inflates its dividend payment will get credited with a higher yield in the financial press, and among the analyst community. It would then seem too hasty to dismiss the dividend payment that appears on the face of the profit and loss account. For one thing, the corporation has a considerable degree of flexibility as to when to raise additional equity when it is over-distributing: it may easily disguise its error by allowing its gearing ratio to creep up over a number of years, only to be offset by a more substantial equity rights issue at some convenient time in the future. As demonstrated later in Part II of the study (Table II-1), allowing the gearing ratio to escalate by 5% or so is unlikely to show up in any material deterioration in its MVA. Secondly, the rights issue would have to appear explicitly as a source in the flow of funds statement.

In fact, the sequence of repurchase plans envisaged in the present study appears to be the one cash flow that Modigliani and Miller (1959) failed to identify as providing the foundation for the corporation's value. They considered discounting various definitions of its earnings, dividends, cash flow and growth opportunities, and were able to demonstrate that each of these various approaches in turn would result in a common fundamental valuation for the company. Of course, they were writing ahead of the various legal and regulatory relaxations that made the stock repurchase mechanism viable, but discounting the annual repurchase amounts defined by the corporation's residual net free cash flows gives the same result also. It is also the case that the internal rate of return achieved by the individual on an investment in the corporation's stock will always be equal to the ECC. This applies equally to those stockholders who sell out in response to the buy-back operation, and those who decide to stay on board for a further period. Of course, this assertion ignores any differences caused by the incidence of the capital gains tax.

Table I-1 shows that the scope available to the management to manipulate the corporation's yield in the way suggested by Modigliani and Miller (1959) depends on the growth rate. For instance, the company growing at 9% has an earnings yield of only 1.752%, compared with a distribution yield of 1.250%: this offers a little flexibility compared with the corporation growing at 6%, with a potential uplift of 4.527% (= 8.777 – 4.250). For the corporation in stasis, the earnings and distribution yields are identical at 10.25%, and this then offers none. The biggest contrast is for the corporation growing at 7%, where the yield difference weighs in at 4.719%. Of course, stockholders are totally immune to these machinations, since they can buy and sell in the marketplace to adjust their cash flows to suit their individual requirements independently of the distribution policy pursued by the corporation's management. But as the table demonstrates, there is an economically significant relationship between the rate of growth and the payout ratio, as discussed in the following section.

VI. A PICTURE OF THE PAYOUT RATIO

Table I-1: Columns and Rows

Table I-1 was constructed by computing simulated sets of accounts for representative corporations growing at rates between 0% and 9% per annum: the relationships underlying the calculations, the WACC and the ROCE, are consistent throughout, as is the size of the assert base. The aim is to demonstrate the factors that influence the corporation's optimal payout policy. The optimal payout policy, and the factors that drive it, are revealed by looking along the rows of the table.

This section and the conclusion that follows will focus on the payout ratio and the degree to which it may indeed be considered optimal. It is then left to Part II of the study to explain its behaviour in terms of the other variables in the table.

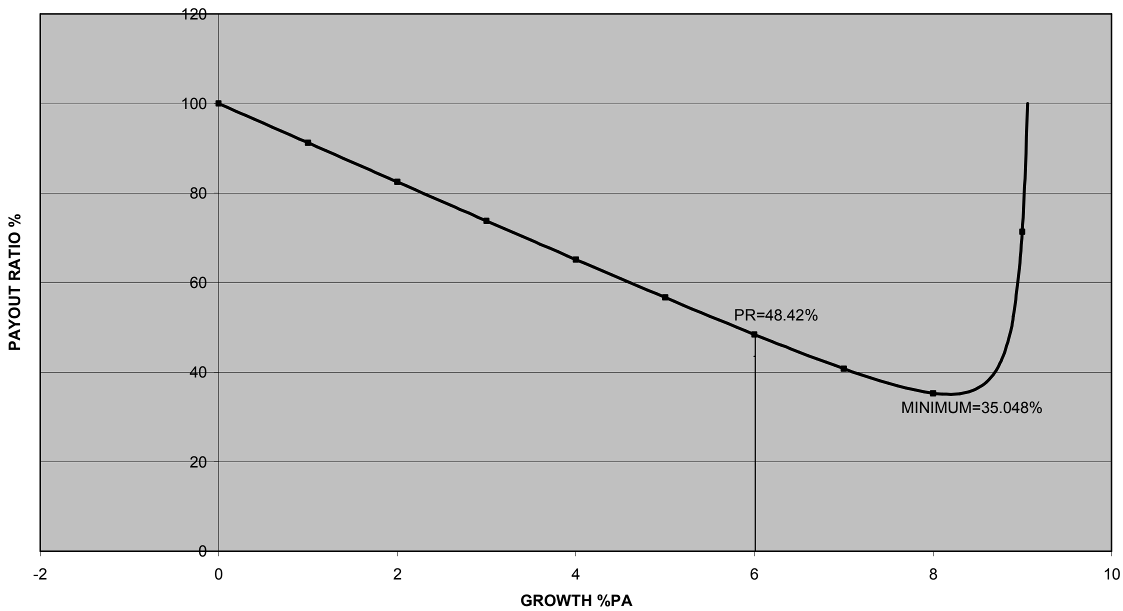
The Payout Formula

The ten points calculated in Table I-1 are depicted as the black squares in Figure I-4 below. The points are set in the context of a mathematical formula for the optimal payout ratio. This is derived firstly from the equation for the dividend introduced in Section II above. The payout ratio may be taken as the ratio of the right-hand side of Equation I-11 to its first two terms. This is the ratio of the amount of the repurchase plan for Period One to the corresponding earnings:

$$PR = 1 - X_1(1 - B_0/A_0)/(A_0 f_1(X_1) - B_0 n(d)) \tag{I-50}$$

where, after some minor manipulation, the payout ratio (PR) is now stated as the difference between unity and the retention ratio. The retention ratio is a more fundamental concept in the scheme ordained by Modigliani and Miller (1959), as it is the ratio of the equity financed portion of the optimal amount of the capital expenditure to the Period One net profit.

FIGURE I-4



Equation I-50 may be simplified by noting that Equations I-6 and I-17 together give:

$$B_0 = dV_0 = dA_0(ROCE - g)/(WACC - g) \quad \text{(I-51)}$$

where $f_1(X_1)$ may, as before, be replaced by the (time-constant) level of the ROCE. After some further manipulation, the payout ratio may be stated as:

$$PR = 1 - g[(W - g) - d(R - g)]/[(R(W - g) - dn(d)(R - g)] \quad \text{(I-52)}$$

where W and R are abbreviations for the WACC and the ROCE respectively. In spite of presenting the equation in this way, its only obvious characteristic is that the retention ratio vanishes where the growth rate is zero, leaving PR = 1, or more usually 100%.

The formula can be given somewhat more shape by substituting in the usual solution values for the WACC, the ROCE, the optimal gearing ratio and the associated level of the (net) interest rate. This results in the power series:

$$PR = (1 - 19.702635g + 97.039389g^2)/(1 - 10.912079g) \quad \text{(I-53)}$$

which has a discontinuity where the denominator vanishes. This occurs for a value of 9.164% (= 100/10.912079) for the growth rate. Fortunately, this is irrelevant in the interpretation of the model, since the payout ratio takes the value unity (100%) for a growth rate of 9.059%: this is the critical value such that the constraint, $B_0 \leq 100$, becomes binding, as described in the previous section. For the record, the payout ratio reaches a minimum of 35.048% at a growth rate of 8.181%, as indicated in Figure 1-4.

Full-Distribution Threshold Rate of Growth

The Harrod–Domar (Domar 1957) model shows that the growth of the economy at the aggregate level will be the product of the savings ratio and the return on capital employed. The corporation proves to be a microcosm of this process: its growth rate will be the product of its savings (retention) ratio (s) and its return on equity (ROE). If it is financed entirely with equity, the ROE will be the same as the ROCE as defined earlier, so that, after rearrangement:

$$s = g/ROCE \qquad \text{(I-54)}$$

In this somewhat simpler version, the retention ratio will be proportional to the growth rate, the gradient being the reciprocal of the net return on capital employed. The payout ratio (= 1 – s) will then slope downwards from an intercept of one (100%), with a gradient equal to the negative of the reciprocal of the ROCE. At the lower growth rates, this downward sloping tendency is clearly observable in Figure I-4. However, as the gap between the ROCE and the ROE opens up along the growth spectrum as a result of the rising level of the indebtedness of the corporations, the relationship is seen to be concave from above. In due course, this effect becomes so powerful that the payout ratio turns upwards and eventually reaches 100% again.

As noted earlier, a lesser gradient in the interest rate function will lower the threshold rate of growth at which the 100% payout ratio becomes applicable. In Equation I-29, that gradient is set at 0.1208. If the gradient were instead 0.102333, the optimal gearing ratio would rise to 30%, the interest rate paid at the net level would rise to 6.53%, and the WACC and the ROCE would fall to 9.169% and 9.9845% respectively. Plugging these values into Equation I-52 shows a minimum for the payout ratio of 42.7% at a 7.6% growth rate, and a return to the 100% level at about 8.7%. A similar effect may be derived by modelling a higher rate of return. In Equation I-35 the intercept for the MEC function takes the value 0.108. If in fact the intercept were greater, the threshold rate of growth at which the 100% payout ratio applies would again be correspondingly lower. It is intended to explore these effects in more detail in a subsequent paper.

Of course, it would be rare to find a company with 100% balance sheet gearing in the quoted sector. Private equity operators are pushing the frontiers in this direction, but it is not known what might be considered an excessive use of the tax shield by the tax authorities. There is already an ongoing debate in the UK concerning the appropriateness of the principle of the tax-deductibility of interest.

VII. THE STORY SO FAR

There remains much to learn about the valuation of the corporation by looking along the rows of Table I-1, and beyond that, about the Trade-Off Theory of Capital Structure by exploring the second order conditions for the maximisation of its market value added as stated in Equation I-18. This sets out the agenda for Part II of the study. At this half-way stage, meanwhile, it is appropriate to sum up the case against the Dividend Irrelevancy Proposition.

Equation I-50 offers a suitable starting point, as the retention ratio – the second term – contains an explicit statement of both the corporation's earnings (the denominator) and the equity component of the financing of its capital expenditure (the numerator) for Period One. Both these amounts appear explicitly in the profit and loss accounts as laid out in Table I-1. The table, as noted previously, defies conventional accounting practice: normally, if there is no intention to pay a cash dividend, the whole of the net profit would be transferred to the corporation's reserves to await allocation either to finance new investment or to finance a stock repurchase operation.

The entries in the table move the argument on from the Modigliani–Miller position only to the extent that they manifest the optimal division of the capital expenditure between that portion which is to be financed by additional borrowing, and that portion which is to be financed with equity. This results from incorporating the Trade-Off Theory of Capital Structure into the model. But, as argued in Section V, this latter amount could still be sourced by the issue of new shares if the corporation decides to apply the whole of its earnings to its repurchase programme each period. However, once the resulting contra entries have been set aside, the net buy-back amounts to the corporation's residual net free cash flow. Nothing appears in the P&L account: but in the flow of funds statement there is an obvious face-off between the excess amount distributed via the repurchase programme (the use) and the amount of equity the corporation needs to raise from its stockholders to finance its optimal capital spend (the source).

It may be recalled that Modigliani and Miller (1959) only arrive at the comparable result – in the statement quoted at the outset of the introduction – by imposing the rather strained assumption that the management of the corporation operates under the self-imposed constraint that it cannot access fresh equity funds.

The case against the Dividend Irrelevancy Proposition is then that it denies the existence of the important formula developed above for the payout ratio (Equation I-52). In theory, this may not appear to matter: the model embraces only two decision variables, and the payout ratio is not one of them. In practice, the formula offers an important new resource for CFOs as they plot their financial strategy, and for investment analysts as they endeavour to assess the level of competence of the managers of the corporations they monitor. The payout formula is optimal in the sense that it follows on logically from an optimisation process. Modigliani and Miller (1959) treat it as defining, for each rate of growth, merely the lower bound of a range of possible payout ratios, up to and including the full distribution of earnings. In their analysis, the management of the corporation can inflate its dividend yield by distributing more than the amount indicated by this minimum, but the compensating equity rights issues then reduce the rate of earnings per share and dividend growth so as to leave the trajectory of its equity market capitalisation through time unaffected. Of course, there is no suggestion that they, nor anyone else, recommend such behaviour.